$$\int \boxed{7x^6}(x^7-800)^{1000}\boxed{dx}$$

Let $u = x^7-800$

$$\frac{du}{dx} = 7x^6$$

$$du = \boxed{7x^6\,dx}$$

$$\int u^{1000}\,du$$

$$= \frac{u^{1001}}{1001} + C$$

$$= \frac{(x^7-800)^{1001}}{1001} + C$$

$$\frac{1}{7}\int 7x^6(x^7-800)^{1000}\,dx$$

Let $u = x^7-800$

$$\frac{du}{dx} = 7x^6$$

$$du = \boxed{7x^6\,dx}$$

$$\frac{du}{7x^6} = dx$$

$$\frac{1}{7}\int u^{1000}\,du$$

$$= \frac{1}{7}\frac{u^{1001}}{1001} + C$$

$$= \frac{1}{7}\frac{(x^7-800)^{1001}}{1001} + C$$

4a. $\int x^2\sqrt{1+x}\,dx$

$u = 1+x$

$\frac{du}{dx} = 1$

$du = dx$

$\int (u-1)^2 (u)^{\frac{1}{2}}\,du$

$\int (u^2-2u+1)u^{\frac{1}{2}}\,du$

$\int \left(u^{\frac{5}{2}}-2u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)du = \frac{u^{\frac{7}{2}}}{\frac{7}{2}} - \frac{2u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

$\frac{2u^{\frac{7}{2}}}{7} - \frac{4}{5}u^{\frac{5}{2}} + \frac{2u^{\frac{3}{2}}}{3} + C$

4.b $\int (\csc(\sin(x)))^2 \cancel{\cos x}\, du$ $u = \sin x$

$\frac{du}{dx} = \cos$

$\frac{du}{\cos x} = dx$

$\int (\csc(\sin(x)))^2 \frac{du}{\cancel{\cos(x)}}$

$\int (\csc(\sin(x)))\, du$

$\int (\csc^2(u))\, du$

$-\cot(u) + C$

$-\cot(\sin(x)) + C$

4.c $\int \sin(x-\pi)\,dx$ $u = x-\pi$

$\int \sin(u)\,du$ $\frac{du}{dx} = 1$

$du = dx$

$-\cos(u)+C$

$-\cos(x-\pi)+C$

4d) $\int \frac{5x^4}{(x^5+1)^2}\,dx$

$u = x^5+1$

$dx \cdot \frac{du}{dx} = 5x^4 \cdot dx$

$\frac{}{5x^4}$

$\int \frac{\cancel{5x^4}}{(u)^2}\left(\frac{du}{\cancel{5x^4}}\right)$ $\quad \frac{du}{5x^4} = dx$

$\int \frac{1}{u^2}\,du$

$\int u^{-2}\,du = \frac{u^{-1}}{-1} + C$ $\quad \frac{(\cancel{x}^5+1)^{-1}}{-1} + C$

5a) $\int \frac{dx}{x\ln x}$; $u = \ln x$ $\quad \int \frac{1}{u} = \ln(u)$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \boxed{\frac{dx}{x}}$$

$$\int \frac{dx}{x(u)} \qquad \int \frac{du}{u} = \ln u + C = \ln \ln x + C$$

5b) $\int e^{-5x}\,dx$; $u = -5x$

$$\frac{du}{dx} = -5$$

$$du = -5dx$$

$$\int e^{u}\,dx \Rightarrow \int e^{u}\,\frac{du}{-5} = -\frac{1}{5}\int e^{u}\,du$$

$$= -\frac{1}{5}\int e^{u}\,du = -\frac{1}{5}e^{u} + C$$

$$-\frac{1}{5}e^{-5x} + C$$

5.c) $\int \frac{\sin 3\theta}{1+\cos 3\theta}\,d\theta$; $u = 1+\cos 3\theta$

$$\frac{du}{d\theta} = -\sin 3\theta \cdot 3$$

$$\frac{du}{-3\sin 3\theta} = d\theta$$

$$\frac{\sin 3\theta}{u}\left(\frac{du}{-3\sin 3\theta}\right) = -\left(\frac{1}{3}\right)\frac{du}{u}$$

$$-\frac{1}{3}\int \frac{du}{u} = \boxed{-\frac{1}{3}\ln|1+\cos(3\theta)| + C}$$

5d) $\int \frac{e^x}{1+e^x}\,dx$; $u = 1+e^x$

$\frac{du}{dx} = e^x$ $\quad du = e^x\,dx$

$\int \frac{du}{u} = \ln(1+e^x)+C$

$\int \frac{1}{u}\,du = \frac{1}{u}\int du = \ln(1+e^x)+C$

$$\int \frac{x^2\,dx}{1+x^6} \qquad \underline{u = x^3} \qquad du = 3x^2\,dx$$

$$\frac{du}{dx} = 3x^2$$

$$\frac{1}{3}\int \frac{3x^2\,dx}{1+x^6} \to \frac{1}{3}\int \frac{du}{1+x^6} \to \int \frac{du}{1+u^2} \to \frac{1}{3}\sin^{-1}x$$

$$u = x^3$$
$$u^2 = x^6$$

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